**QuantLib User Meeting Presentation:**

**Intro – Table of contents (slide 1/2):**

Good morning everyone,

It’s a big pleasure for me to be here and present this work named: “Advanced EONIA Curve Calibration” performed with Ferdinando Ametrano and Paolo Mazzocchi.

This work analyses subtle but relevant issues related to EONIA curve calibration in order to propose solutions able to improve the outcome quality.

For each of the topics treated, which are shown at points 1,2 and 3 in the table of contents, we have performed

- a market empirical analysis

- an Excel implementation using the latest QuantLib version

- a solutions’ validation using a repricing error analysis.

**Intro jump (slide 3):**

So, let’s start from the first issue which in related to jumps and turn-of-year effect in the EUR market with a brief focus also on the U.S market case.

**Jump empirical evidence (slide 4):**

One of the key point to obtain a state-of-art overnight curve, is to obtain smooth forward rates.

For even the best interpolation scheme to be effective, before calibration starting it is very important to remove all the estimated market rate jumps

and then add them back at the end of the process.

The most relevant rate jump is the so called turn-of-year effect

which can be seen in the last working day in market quotations spanning across the end of the year.

In a financial point of view, a rate jump is an higher index fixing due to the increased search of liquidity of market participants due to end-of-month or end-of-year capital requirements.

**Figure jump EUR (slide 5):**

The figure shows the 2014 EONIA index time series

where it is visible the December turn of year effect

which, that year, has been of particular big size.

**Figure jump USD (slide 6):**

However, the previous definition is not consistent for the U.S. market case

because the Fed Funds rate time series (which is the equivalent of the overnight rate in U.S. market) shows negative jumps at least every end-of-month as visible in this Bloomberg terminal screenshot. (FINE)

**Jump estimation (slide 7): (ANIMAZIONE)**

In order to estimate jumps sizes, the suggestion is to use a 4-step methodology inspired by Burghardt and proposed by Ametrano Mazzocchi:

1. So, First of all we have to calibrate an overnight curve using a flat interpolator and including all liquid market quotes available
2. Once the curve is calibrated, estimate the first jump assuming that a segment out of line with preceding and following segments can be put back in line dumping the difference into the jump size; So, Positive jump can be estimated using the following formula
3. 4) Once the first jump has been estimated we can remove it from the curve and then repeat point 2 and 3 on the next jump date.

**Jump estimation (slide 8):**

The formula presented is good for positive jump estimation.

But, In order to account also negative jumps, as the one shown in the U.S. market case, it must be reviewed as visible on the slide.

The only difference is that you have to reverse the order of interpolated forward and original forward rate

**Figure Eur Curve (slide 9):**

The next figures show examples of OVERNIGHT curves calibrated using the approach described before.

So, we estimate jumps’ sizes,

we remove them before calibration

and then we add them back at the end of the process.

This is the resulting EONIA curve which accounts the first jump and turn-of-year effect.

**Figure Usd Curve (slide 10):**

This other figure, instead, shows a USD overnight curve which accounts all the estimated jumps.

As visible our approach estimates positive jumps for the EUR market case and negative jumps for the USD market case

which is consistent with the empirical evidence.

**Intro Forward Stub (slide 11):**

So, first issue has been presented, let’s skip to the second point.

**Imperfect concatenation (slide 12):**

This problem concerns the EONIA curve composition which instruments’ selection best practice is to mix spot starting instruments (the Overnight Indexed Swaps) and forward starting instruments (namely the European Central Bank overnight indexed swaps).

Since this ECB OIS are preferred by traders because of their liquidity, the calibration algorithm is set for giving a major priority to this instruments despite of spot starting OIS which covers the same period.

As a consequence, the EONIA curve will show an overlapping section caused by the imperfect concatenation between spot and forward instruments.

**Figure Piecewise Constant (slide 13):**

The next figure show the last year EONIA index fixings

As visible, they have a piecewise constant behaviour between ECB monetary policy dates.

It is important to underlying that 3 December 2015 and 10 March 2016 were both ECB dates.

I want to stress this point because this market evidence it’s a backbone for the analysis that I’m going to present later.

**Figure Overlapping IMP (slide 14):**

In this figure, which is related to a dataset dating back to the last January, the evidence of overlapping instruments in clearly shown.

The blue line is the one month maturity spot OIS

Which market value fixes the average rate from spot date to 1 month;

The dotted line is the two months’ maturity OIS

And it fixes the average rate in the interval (1M,2M)

the green and violet lines are the first two ECB OIS quoted by the market.

In particular the first ECB OIS (the green line) fixes the average rate for the interval (2M,ECBend)

It is visible how, in this scenario, the two months’ maturity OIS overlaps the first ECB OIS for the period which goes from the first ECB OIS settlement and the OIS2M maturity.

**Imperfect concatenation (slide 15):**

So, since the bootstrap algorithm uses this information to perform a perfect re-price of all the included market quotes,

it is not able to derive the information set by the first ECB OIS for the interval (ECBstart; 2M) because it uses the level fixed by OIS2M.

This means that the calibrator is not able to use the information set by the most liquid instrument on the market.

**Solution (slide 16):**

The consequent distortion could be negligible only if the information given by both instruments are almost equal

as, for example, when the curve has a flat behaviour in that region.

Otherwise, there are situations where the level fixed by these two instruments is very different as, for example, when first ECB OIS is accounting a rates cut/rise expectation.

In order to solve this problem our suggestion is to build a Meta-Quote called: “Forward Stub”

with settlement date equal to the maturity of the last not-overlapping OIS and maturity date equal to the settlement of the first ECB OIS quoted by the market.

The result is a new instrument which links perfectly spot starting OIS to forward starting ECB OIS.

**Solution (slide 17): (ANIMAZIONE)**

Of course, The Forward Stub value, is not quoted in the market but it is implied in the market and can be derived imposing a no-arbitrage condition.

The condition, which is show on the slide, must always ensure the perfect reprice of the overlapping spot instrument.

Analysing this formula we know that:

The instantaneous forward rate integral from 0 (spot date) to 1M is known and equal to the OIS1M value quoted by the market.

The integral from 1M, which is the last not-overlapping spot OIS maturity, to the first ECB OIS settlement is the unknown Forward Stub value that we want to derive.

The integral from the first ECB OIS settlement to 2M it is not properly a quoted instrument

And finally, the integral from 0 (spot date) to 2M is known and equal to the OIS2M which is the overlapping instrument.

So, in this situation we have two unknown to define. To find a solution we have to make an assumption in order to define the value of the integral from ECBstart to 2M

**Solution (slide 18):**

To have information related to the interval (ECBstart,2M) we need to assume flat forward rates in the interval (ECBstart, ECBend) which are both European Central Bank monetary policy meeting dates.

So, we are assuming piecewise constant rates between ECB monetary policy dates and we already show that this assumption is supported by market evidence.

With this assumption we can fix the average rate in the interval (ECBstart,2M) equal to the average rate in the interval (ECBstart, ECBend)

Which, obviously, is a known value equal to the first ECB OIS market quote.

So, what we suggest is to exclude from the calibration the overlapping instrument and replace it with this new Meta-Quote

which is able to link spot to forward instruments and which value will always ensure the perfect reprice of the discarded spot OIS. (FINE SLIDE)

**Solution (slide 19):**

Now all the values necessary to derive the Forward Stub quote are well-known

and the problem is reduced to a simple equation with only one unknown value

that can be calculated, assuming simple compounding, with the formula shown on the slide.

**Solution (slide 20):**

In next figure we can see the same scenario shown previously but applying the Forward Stub solution.

It is visible that the Meta-Quote links perfectly spot to forward instruments avoiding unwanted distortion and shape oscillation.

As a consequence the overlapping instrument , the OIS2M, will be excluded from the calibration.

**Solution (slide 21):**

Another important point to underlying, which is a limit case, is that,

since a generic spot OIS with x-maturity always cover the year fraction from today to today+x (where x can be years, months, weeks), the ECB OIS settlement is fixed at a specific date which every day becomes earlier.

Since the first ECB OIS settlement becomes earlier and earlier, it will happen that the discarded spot instrument is the spot week OIS.

This case implies that the so called “Forward Stub” it is not forward start anymore because it covers the period which goes from Spot Date to the first ECB OIS settlement date.

So, the “Spot Stub” implied quoted can be derived using the following formula which is a particular case of the preceding one.

**Repricing errors (slide 22):**

To validate our solution we present now a repricing error analysis.

This table summarize the differences between ICAP market quotes and the repricing performed with both curves

namely the one with overlapping instruments and the one which include the forward stub.

The instruments’ prices are calculated for all not Included market quotes which are listed in the first two columns.

Keep in mind that while included instruments are perfectly re-priced by construction, all not included instruments show re-pricing errors.

Our solution validation mostly consists in minimizing this errors.

As you can see, our solution is able to minimize all not included instruments’ re-pricing errors and so it produces a more accurate curve.

In some cases, errors are minimized of over 1 bps as,

for example, for the OIS3M which is a relevant difference especially for trading purpose.

Furthermore, we can see that the OIS2M is perfectly repriced in both cases.

The difference is that, in the overlapping case it is an included instrument perfectly re-priced by construction

while, in the forward stub case, it is re-priced perfectly due to the no-arbitrage condition set before.

Errors are also summarized with a Root Mean Square Error and a Maximum error which are obviously higher in the overlapping case.

**Figure Curve overlap (slide 23):**

The next figure shows an example of EONIA curve, bootstrapped with overlapping instruments.

It is visible how the calibrator, which is not using the most reliable information given by the first ECB OIS

Produces a strange behaviour in order to reprice perfectly all the included instruments.

**Figure Curve forward stub (slide 24):**

This other figure, instead, shows an example of EONIA curve, bootstrapped including the forward stub.

It is straightforward that this solution avoids distortion and strange oscillations.

**Intro Mixed (slide 25):**

So, also the second point has been presented. Let’s now discuss the last issue which is related to the EONIA curve functional form.

**EONIA functional form (slide 26):**

I already sad that EONIA Index fixings present a particular behaviour;

in fact, empirical evidence shows that they tend to be piecewise constant between ECB monetary policy dates.

Otherwise, assuming flat forward rates in the mid-long term would be a big inconsistency and would lead to very high repricing errors.

Modelling the EONIA curve requires to fit all the shape’s characteristics

and the problem is reduced to the choice of the best interpolation scheme able to reproduce the curve’s behaviour.

**EURON functional form (slide 27):**

So, in order to reproduce a piecewise flat behaviour, a log-linear interpolation would be a good fit.

But, obviously, it is not advisable to interpolate log-linearly the whole curve;

it would produce big inconsistency because in the mid-long term we have a decreasing instruments’ granularity.

For this reason a linear interpolation for the whole curve would lead to very long flat rates period which are not consistent.

As a consequence, the best practice to interpolate overnight related curves converges towards cubic interpolation schemes, which are able to produce smooth forward rates.

The problem occurs since this kind of interpolations don’t have a good fit with the short-term curves section (which, as sad before, is piecewise constant).

Furthermore, using cubic interpolations, the forward stub algorithm presented before would not be consistent anymore

since it assumes a piecewise constant behaviour between ECB dates.

However, we will see that the implied Meta-Quote could be calculated anyway by means of root finding methods.

**Solution (slide 28):**

To fix this interpolation problem we suggest to build a new interpolation scheme named: “Mixed-Interpolation” that gives the possibility to merge two different interpolation techniques.

The concept is very simple but in practice has some critical issues to be fixed.

In particular, we don’t have information about what is the best point where the interpolation schemes must be switched.

Furthermore, we don’t even know which is the best merging approach to be used.

**Solution (slide 29):**

Our suggestion is to merge a log-linear interpolation up to the end of the ECB OIS strip quoted on the market

and then switch to a monotone cubic Hyman filtered interpolation till the end of the curve.

The so called: “Switch Pillar”, which is the pillar where the interpolation technique is changed,

is set equal to the maturity of the last quoted ECB OIS.

This solution fits well the short-term section due to the log-linear interpolator

but also fits well the mid-long term section due to the smoothness produced by the cubic filtered scheme.

**Quantilib implementation (slide 30): (ANIMAZIONE)**

The Mixed Interpolation is already implemented in QuantLib 1.9 version in two different ways:

The first approach is called Split Range Mixed Interpolation.

It consists in interpolating till the switch pillar using the first interpolation

and then switch to the second one for the rest of the curve.

The second approach, which is named ShareRange, consists in interpolating two times the whole curve:

first time with the first interpolation chosen and second time with the other one.

Once both curves have been calibrated the mixed interpolated curve is built linking the obtained curves in the switch pillar.

We will see later that there is no significant difference between this approaches;

In fact, the short term section is the same in both cases,

the only difference could be seen in the cubic interpolated section because cubic interpolations are global schemes and

For this reason, the mid-long term section could be different because it is influenced by the short-term section.

**Repricing error analysis (slide 31):**

The next figure shows the same repricing error analysis presented for the previous point.

The instruments repriced are the same and they are summarized in the first two columns.

The table compares 4 different possibilities:

* a whole linear interpolated curve which includes the Forward Stub (and so without overlapping instruments) which errors are summarized in the first white column.
* The next one is a whole monotone cubic Hyman filtered curve which also include the Meta-Quote that is not calculated with our algorithm.

But using a root finding process.

The related errors are summarized in the second white column.

* The next two settings are two mixed interpolated curves using both QuantLib approach (so, Splitrange approach for the third column and Sharerange approach for the last column).

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As visible there is no significant difference between Splitrange and Sharerange approach

and both, obviously, leads to the same errors as the ones shown by the linear approach.

The most important information is given by the comparison between the whole linear and the whole monotone cubic curves;

as visible the linear interpolated curve shows lower errors for all instruments.

As a consequence the analysis confirms that the linear interpolation fits the short term section better than the cubic one.

This result confirms that the mixed “linear-cubic” interpolation is the one able to minimize errors

and so is the one which lead to a better fit of the overnight curve’s behaviour.

**Figure mixed curve (slide 32):**

The next figure show an example of EONIA curve obtained with our solution,

so, mixing a log linear interpolator till the end of the ECB OIS strip

and a monotone cubic Hyman filtered interpolation till the end of the curve.

It is visible that the curve’s behaviour is piecewise constant and then start to be smooth due to the cubic scheme.

**Intro conclusion (slide 33):**

Summing up, we have presented solutions able to improve EONIA curve calibration.

**Conclusions (slide 34/35/36): (ANIMAZIONE)**

First of all, we propose a 4 step approach to estimate jumps sizes for both EUR and USD market.

Then we suggest to link spot to forward instruments building a new Meta-Quote called Forward stub in order to avoid shape oscillation.

Finally, we suggest to use a new interpolation technique which merges a linear interpolator for the short them section and a cubic interpolator for the rest of the curve in order to reach a better fit.

MY PRESENTATION IS FINISHED….

THANK YOU VERY MUCH FOR LISTENING….